

5) a) Let  $x=2$ ,  $y=2$

$$xy = 4 \geq 1 \text{ and thus } (2,2) \in R.$$

Therefore, not irreflexible.

b)  $x$  and  $y$  will never be the same value.

Since  $(x,x)$  or  $(y,y)$  do not exist, they can never be  $\in R$ . Therefore, it is irreflexible.

c) Given any  $a \in A$ , we have  $(a,a) \in R$  and  $(a,a) \in S$ .

The relation  $R-S$  becomes  $\{(x,y) : x,y \in A \wedge (x,y) \in R \wedge (x,y) \notin S\}$

Since both  $R$  and  $S$  are reflexive, there is no pair  $(x,y)$  that we can eliminate and thus there is no pair in  $R-S$ , making it irreflexible.

6) a) Due to the fact that  $R$  is transitive,

$$R^2 \subset R \text{ and thus } R^n \subset R \text{ (induction) } \dots R^+ = \bigcup_n R^n = R$$

$$R^* = R = \{(x,y) : y-x \geq 0\}$$

b) For  $R^2$ , we do  $R \circ R = \{(x,z) : \exists y (y-x \geq 1 \wedge (z-y) \geq 1)\}$

$$R^2 = \{(x,z) : z-x \geq 2\}$$

$$\text{Now by induction, we get } R^n = \{(x,z) : z-x \geq n\}$$

$$7) R^* = \{(a,a), (a,c), (b,b), (b,d), (c,a), (c,c), (d,b), (d,d), (e,b), (e,d)\}$$

8) If we choose  $R$  to be  $\{(1,2), (3,2)\}$  on  $1,2,3$

$$\text{Transitive clos} = \{(1,2), (3,2)\}$$

$$\text{Reflexive clos} = \{(1,1), (2,2), (3,3), (1,2), (3,2)\}$$

$$\text{Symmetric clos} = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3)\}$$

$(1,3)$  is not part of the set and so the conclusion is not transitive. Since it is not transitive, the conditions for an equivalence relation are not satisfied.